# Exam (6 ECTS) 

## Advanced Public Finance

## Wirtschaftswissenschaftliche Fakultät der Friedrich-Schiller-Universität Jena Prof. Dr. Silke Übelmesser

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| First name: |  | Last name: |  |
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| Student ID: |  | Study programme: |  |

## Please note:

(a) The exam consists of 9 pages including this one. Please check whether your copy of the exam is complete.
(b) The exam consists of 3 questions. The maximum number of points is 60 . You have 60 minutes to complete the exam.
(c) Please answer the questions by writing into the boxes provided after each question. Do not use your own paper! Fill your name and student ID number into the form at the top of each page.
(d) If not defined otherwise, variables have the same meaning as in class. Please make sure that your answers are clearly legible and without any ambiguity. Your answers have to be tractable. If you use diagrams, make sure to label and explain them.
(e) You may use a calculator, but it must not have a text storage function. You may use a dictionary, but it must not contain any notes.
(f) It is your own responsibility to hand in your copy of the exam to the supervisory staff at the end of the exam.

| Question | 1 | 2 | 3 | Sum | Grade |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Max. no. of points | 20 | 20 | 20 | 60 |  |
| No. of points received |  |  |  |  |  |

## Question 1: Non-Linear Income Taxation (20 Points)

Consider an economy with two types of households $i,(i=H, L)$, one that has high ability $(H)$ and the other with low ability $(L)$. Each household $i$ has the separable utility function given by (1). Income is defined as in (2). The government can observe consumption $x^{i}$ and income $y^{i}$, but wage $w^{i}$ and hours worked $A^{i}$ cannot be directly observed.

$$
\begin{gather*}
U^{i}=u\left(x^{i}\right)+v\left(A^{i}\right) \quad u^{\prime}>0, u^{\prime \prime}<0, v^{\prime}<0, v^{\prime \prime}<0  \tag{1}\\
y^{i}=w^{i} A^{i} \tag{2}
\end{gather*}
$$

1.1. What would the term $\left.\frac{d x^{i}}{d y^{i}}\right|_{\bar{U}}$ represent, and how could it be interpreted? (3 Points)

For a given consumption-labor supply pair $(x ; y)$, it holds that

$$
\begin{equation*}
\left.\frac{d x^{L}}{d y^{L}}\right|_{\bar{U}}>\left.\frac{d x^{H}}{d y^{H}}\right|_{\bar{U}} \tag{3}
\end{equation*}
$$

1.2. How is this type of agent preference be called? What does this mean? (4 Points)

The government maximizes the utility of the high-ability household under the constraint that the low-ability household reaches at least a given utility level. The maximization problem of the government is thus given by (4), with the constraints (5), (6), (7) and (8). $N^{i}$ represents the number of households of type $i$.

$$
\begin{equation*}
\max _{\left(x^{H}, x^{L}, y^{H}, y^{L}\right)} U^{H}\left(x^{H} ; y^{H}\right) \tag{4}
\end{equation*}
$$

s.t.

$$
\begin{align*}
U^{L}\left(x^{L} ; y^{L}\right) & \geq \bar{U}^{L}  \tag{5}\\
U^{H}\left(x^{H} ; y^{H}\right) & \geq U^{H}\left(x^{L} ; y^{L}\right)  \tag{6}\\
U^{L}\left(x^{L} ; y^{L}\right) & \geq U^{L}\left(x^{H} ; y^{H}\right)  \tag{7}\\
\left(y^{H}-x^{H}\right) N^{H}+\left(y^{L}-x^{L}\right) N^{L} & \geq 0 \tag{8}
\end{align*}
$$

1.3. Provide an interpretation for each of the constraints. In class, we have not considered one of the four constraints. Explain which one and why. (8 Points)

A Pareto-optimal tax system should be characterized by "No distortions at the top". Use the following graph to answer the questions below.

1.4. What does the vertical distance between the budget line and the indifference curve of the high-productive household represent? (1 Point)
1.5. Name one of the points which presents a non-Pareto optimal allocation and explain why? (2 Points)
1.6. What is the result for the low-productive household? (2 Points)

## Question 2: Private Provision of Pure Public Goods (20 Points)

Consider an economy in which there are $H$ households $(H \geq 2)$, with $h=1, \ldots, H$, who contribute to provide a quantity $G$ of a public good. The contribution of household $h$ to the provision of the public good is denoted by $g^{h}$; whereas $x^{h}$ denotes the consumption of the private good by household $h$. The income of household $h$ is represented by $w^{h}$.
The household maximizes the utility function (9), subject to the constraints given by (10), (11), and (12); with the assumption that all prices are normalized to 1 .

$$
\begin{gather*}
\max _{x^{h}, g^{h}} U^{h}=x^{h} G  \tag{9}\\
w^{h}=x^{h}+g^{h}  \tag{10}\\
G=g^{h}+G^{-h} \quad \text { where } \quad G^{-h}=\sum_{j \neq h} g^{j}  \tag{11}\\
g^{h} \geq 0 \tag{12}
\end{gather*}
$$

2.1. Explain the economic intuition of the constraints (10), (11), and (12). (3 Points)
2.2. Use the constraints (10), (11), and (12) to write down the maximization problem of household $h$. (Do not solve it!) (2 Points)

The reaction function following from the household's maximization problem is:

$$
\begin{equation*}
g^{h}=\max \left(\frac{w^{h}-G^{-h}}{2} ; 0\right) \tag{13}
\end{equation*}
$$

2.3. Derive the reaction function (13) from the household's maximization problem you found in question 2.2. (5 Points)
$\square$

In a symmetric equilibrium, the total private contribution to the provision of the public good is given by (14). The socially optimal contribution to the public good is represented by (15).

$$
\begin{gather*}
G^{\text {priv }}=\frac{H}{H+1} w^{h}  \tag{14}\\
G^{\text {opt }}=\frac{H w^{h}}{2} \tag{15}
\end{gather*}
$$

2.4. Use (14) and (15) to explain that the private provision of public goods leads to a market failure. (2 Points)
2.5. What would happen for the case where $H=1$ ? and for larger values of $H$ ? (2 Points)
$\square$
2.6. Explain the economic intuition behind this market failure.(6 Points)

## Question 3: Multiple Choice Questions (20 Points)

You will be awarded one point for ticking a correct statement and for not ticking an incorrect statement. You will neither receive nor lose points for marking statements incorrectly.

Question 3(a) Tax Evasion (5 Points)
Correct?
Tax evasion is the intentional failure to declare taxable economic activity, while the illegal reorganization of economic activity is called tax avoidance.

| $\square$ | With decreasing absolute risk aversion, higher income leads to a higher ab- <br> solute level of tax evasion. The reason is that higher income then makes <br> individuals less risk averse. |
| :---: | :--- |
| $\square$ | Increasing the probability of detection or increasing the gross fine rate would <br> lead to a reduction in declared income. |
| $\square$ | Increasing the gross fine rate has positive marginal costs of auditing. |
| $\square \quad$ | The optimal policy implies zero tax evasion in equilibrium. |

Question 3(b) Impure Public Goods (5 Points)
Correct?
With an impure public good, there is rivalry with many users/a high intensity of usage, which reduces the return the public good gives to each user.

| $\square \quad$In making his choice, an individual does not take into account the costs <br> resulting from the use of the good by others. |
| :---: | :--- |
| $\square \quad$To internalize the congestion externality, a user charge equal to this externa- <br> lity (at the point of optimal usage) has to be implemented. This user charge <br> is also sufficient to finance the impure public good. |
| $\square \quad$With increasing returns to scale, $\lambda<0$, the capital tax revenues are not <br> enough to cover the costs of providing an optimal amount of infrastructure. |
| $\square \quad$The Selection Principle implies that the government limits itself to the provi- <br> sion of those public goods for which $\lambda>0$, i.e. when the efficient congestion <br> charge is not sufficient to finance the cost of infrastructure. |

Question 3(c) Pure Public Goods (5 Points)
Correct?
The Samuelson Rule says: The marginal rate of substitution between a private and a public good has to be equal to the sum of the marginal rates of transformation.

| $\square \quad$When a public good is financed by a distorting commodity taxation, there <br> is overprovision compared to a non-distorting tax-financing. |
| :---: | :--- |
| $\square \quad$When the public good is privately provided, the quantity of the public good <br> falls short of the socially optimal quantity. Only in the limiting case with <br> one individual are both quantities the same. |

$\square \quad$ In order to learn the individual preferences about the public good, an interview is an incentive compatible mechanism.
$\square \quad$ The idea of the Lindahl mechanism is to charge each consumer an individual or personalized price for the good reflecting individual preferences. This is, however, not incentive compatible.

Question 3(d) Public Provision of Private Goods (5 Points)
The paper by Grunow and Nuscheler (2014) makes a switcher analysis based on a sample with two types of individuals: one type that is allowed to switch from one health system to another, and a second type that is not.
Correct?
Passive risk selection refers to the insurers' ability to screen out the healthy from the pool of those who are allowed to switch.

| $\square$ | The private insurers charge risk-based premiums while the public insurers <br> (sickness funds) charge income-based premiums. |
| :---: | :--- |
| $\square$ | Individuals with children have an incentive to be publicly insured, ceteris <br> paribus. |
| $\square$ | In general, to provide private goods publicly in an efficient way; it is required <br> that an inverse Samuelson Rule is followed. |
| $\square \quad$Privately insured individuals that have experienced a health shock have a <br> higher probability to switch from private to public health insurance as com- <br> pared to those without such a shock. |  |

